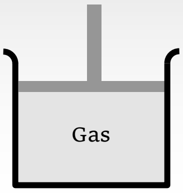


Chapter 13

Thermodynamic Processes

CONTENTS

Watt James, a Scottish inventor and mechanical engineer is renowned for his improvements of the steam engine. Watt was born on January 19, 1736, in Greenock, Scotland.

Watt determined the properties of steam, especially the relation of its density to its temperature and pressure, and designed a separate condensing chamber for the steam engine that prevented enormous losses of steam in the cylinder and enhanced the vacuum conditions. Watt's first patent, in 1769, covered this device and other improvements on Newcomen's engine, such as steam-jacketing, oil lubrication, and insulation of the cylinder in order to maintain the high temperatures necessary for maximum efficiency.

Thermodynamic Processes

13.1 Introduction

(1) **Thermodynamics :** It is a branch of science which deals with exchange of heat energy between bodies and conversion of the heat energy into mechanical energy and vice-versa.

(2) **Thermodynamic system :** A collection of an extremely large number of atoms or molecules confined with in certain boundaries such that it has a certain value of pressure, volume and temperature is called a thermodynamic system. Anything outside the thermodynamic system to which energy or matter is exchanged is called its surroundings.

Example : Gas enclosed in a cylinder fitted with a piston forms the thermodynamic system but the atmospheric air around the cylinder, movable piston, burner *etc.* are all the surroundings.

Thermodynamic system may be of three types

(i) Open system : It exchange both energy and matter with the surrounding.

(ii) Closed system : It exchange only energy (not matter) with the surroundings.

(iii) Isolated system : It exchange neither energy nor matter with the surrounding.

(3) **Thermodynamic variables and equation of state :** A thermodynamic system can be described by specifying its pressure, volume, temperature, internal energy and the number of moles. These parameters are called thermodynamic variables. The relation between the thermodynamic variables (P, V, T) of the system is called equation of state.

For μ moles of an ideal gas, equation of state is $PV = \mu RT$ and for 1 mole of an it ideal gas is $PV = RT$

For μ moles of a real gas, equation of state is $\left(P+\frac{a\mu^2}{V^2}\right)(V-\mu b)=\mu RT$ and for 1 mole of a real gas it is $\left(P+\frac{a}{V^2}\right)(V-b)=RT$

(4) **Thermodynamic equilibrium :** When the thermodynamic variables attain a steady value *i.e.* they are independent of time, the system is said to be in the state of thermodynamic equilibrium. For a system to be in thermodynamic equilibrium, the following conditions must be fulfilled.

(i) Mechanical equilibrium : There is no unbalanced force between the system and its surroundings.

(ii) Thermal equilibrium : There is a uniform temperature in all parts of the system and is same as that of surrounding.

(iii) Chemical equilibrium : There is a uniform chemical composition through out the system and the surrounding.

(5) **Thermodynamic process :** The process of change of state of a system involves change of thermodynamic variables such as pressure *P*, volume *V* and temperature *T* of the system. The process is known as thermodynamic process. Some important processes are

(i) Isothermal process (ii) Adiabatic process (iii) Isobaric process (iv) Isochoric (isovolumic process)

(v) Cyclic and non-cyclic process (vi) Reversible and irreversible process

13.2 Zeroth Law of Thermodynamics

If systems *A* and *B* are each in thermal equilibrium with a third system *C*, then *A* and *B* are in thermal equilibrium with each other.

(1) The zeroth law leads to the concept of temperature. All bodies in thermal equilibrium must have a common property which has the same value for all of them. This property is called the temperature.

(2) The zeroth law came to light long after the first and seconds laws of thermodynamics had been discovered and numbered. Because the concept of temperature is fundamental to those two laws, the law that establishes temperature as a valid concept should have the lowest number. Hence it is called zeroth law.

13.3 Quantities Involved in First Law of Thermodynamics

(1) **Heat (ΔQ) :** It is the energy that is transferred between a system and its environment because of the temperature difference between them. Heat always flow from a body at higher temperature to a body at lower temperature till their temperatures become equal.

Important points

(i) Heat is a form of energy so it is a scalar quantity with dimension $[ML^2T^{-2}]$.

(ii) Unit : *Joule* (S.I.), *Calorie* (practical unit) and 1 *calorie* = 4.2 *Joule*

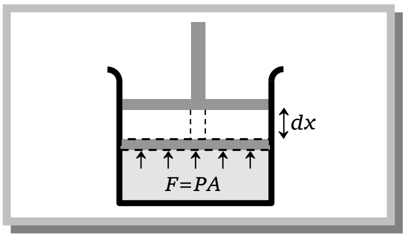
(iii) Heat is a path dependent quantity *e.g.* Heat required to change the temperature of a given gas at a constant pressure is different from that required to change the temperature of same gas through same amount at constant volume.

(iv) For solids and liquids : $\Delta Q = mL$ [For change in state] and $\Delta Q = mc\Delta T$ [For change in temperature]

For gases when heat is absorbed and temperature changes :

$(\Delta Q)_V = \mu C_V \Delta T$ [For constant volume] and $(\Delta Q)_P = \mu C_P \Delta T$ [For constant pressure]

(2) **Work (ΔW) :** Work can be defined as the energy that is transferred from one body to the other owing to a force that acts between them



If P be the pressure of the gas in the cylinder, then force exerted by the gas on the piston of the cylinder $F = PA$

In a small displacement of piston through dx, work done by the gas

$$dW = F.dx = PA\,dx = P\,dV$$

$\therefore$ Total amount of work done $\Delta W = \int dW = \int_{Vi}^{V_f} P\,dV = P(V_f - V_i)$

Important points

(i) Like heat, work is also a path dependent, scalar physical quantity with dimension $[ML^2T^{-2}]$

(ii) From $\Delta W = P\Delta V = P(V_f - V_i)$

ΔW = positive if $V_f > V_i$ *i.e.* system expands against some external force.

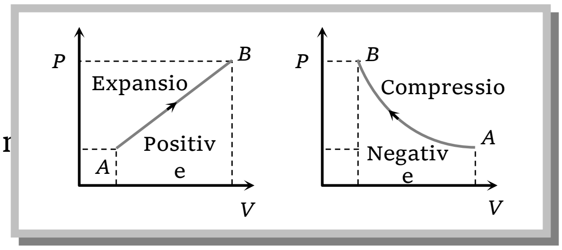
ΔW = negative if $V_f < V_i$ *i.e.* system contracts because of some external force exerted by the surrounding.

(iii) In *P-V* diagram or indicator diagram, the area under *P-V* curve represents work done.

W = area under *P-V* diagram

It is positive if volume increases (for expansion)

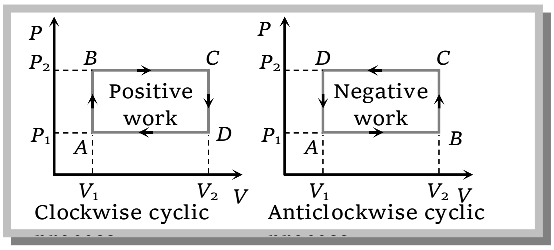
It is negative if volume decreases (for compressio



(iv) In a cyclic process work done is equal to the area under the cycle.

It is positive if the cycle is clockwise.

It is negative if the cycle is anticlockwise.



(v) $W = \int_{V_i}^{V_f} P\,dV$

From this equation it seems as if work done can be calculated only when *P-V* equation is known and limits V_i and V_f are known to us. But it is not so. We can calculate work done if we know the limits of temperature.

For example, the temperature of n moles of an ideal gas is increased from T_0 to $2T_0$ through a process $P=\frac{\alpha}{T}$ and we are interested in finding the work done by the gas. Then

$$PV = nRT \qquad \text{(ideal gas equation)} \qquad .....\text{(i)}$$

and $$P=\frac{\alpha}{T} \qquad .....\text{(ii)}$$

Dividing (i) by (ii), we get $V=\frac{nRT^2}{\alpha}$ or $dV=\frac{2nRT}{\alpha}dT$

$$\therefore \qquad W=\int_{V_i}^{V_f}P\,dV=\int_{T_0}^{2T_0}\left(\frac{\alpha}{T}\right)\left(\frac{2nRT}{\alpha}\right)dT=2nRT_0$$

So we have found the work done without putting the limits of volume.

(vi) If mass less piston is attached to a spring of force constant K and a mass m is placed over the piston. If the external pressure is P_0 and due to expansion of gas the piston moves up through a distance x then

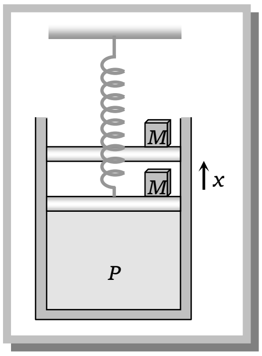
Total work done by the gas $W=W_1+W_2+W_3$

where W_1 = Work done against external pressure (P_0)

W_2 = Work done against spring force (Kx)

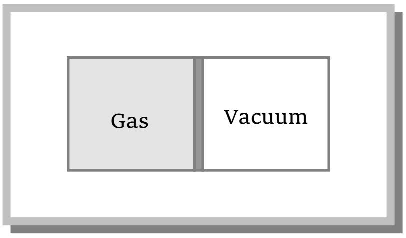
W_3 = Work done against gravitational force (mg)

$$\therefore \qquad W=P_0V+\frac{1}{2}Kx^2+mgx$$



(vii) If the gas expands in such a way that other side of the piston is vacuum then work done by the gas will be zero

As $W=P\Delta V=0$ [Here P = 0]



(3) **Internal energy (U) :** Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration.

The energy due to molecular motion is called internal kinetic energy U_K and that due to molecular configuration is called internal potential energy U_P.

i.e. Total internal energy $U=U_K+U_P$

(i) For an ideal gas, as there is no molecular attraction $U_p=0$

i.e. internal energy of an ideal gas is totally kinetic and is given by $U = U_K = \frac{3}{2}\mu RT$

and change in internal energy $\Delta U = \frac{3}{2}\mu R\,\Delta T$

(ii) In case of gases whatever be the process

$$\Delta U = \mu\frac{f}{2}R\Delta T = \mu C_V\Delta T = \mu\frac{R}{(\gamma-1)}\Delta T = \frac{\mu R(T_f - T_i)}{\gamma-1} = \frac{\mu RT_f - \mu RT_i}{\gamma-1} = \frac{(P_fV_f - P_iV_i)}{\gamma-1}$$

(iii) Change in internal energy does not depends on the path of the process. So it is called *a* point function *i.e.* it depends only on the initial and final states of the system, *i.e.* $\Delta U = U_f - U_i$

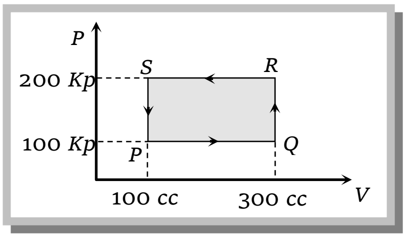
(iv) Change in internal energy in a cyclic process is always zero as for cyclic process $U_f = U_i$

So $\Delta U = U_f - U_i = 0$

Sample problems based on ΔQ, ΔU and ΔW

Problem **1.** A thermodynamic system is taken through the cycle *PQRSP* process. The net work done by the system is

[Orissa JEE 2002]



(a) 20 *J*

(b) – 20 *J*

(c) 400 *J*

(d) – 374 *J*

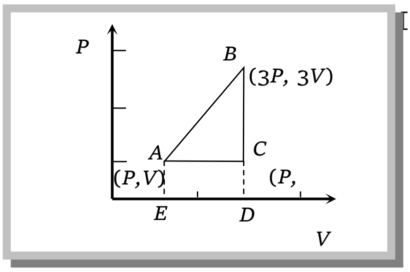
Solution : (b) Work done by the system = Area of shaded portion on *P*-*V* diagram

$$= (300-100)10^{-6}\times(200-10)\times10^{3} = 20\ J$$

and direction of process is anticlockwise so work done will be negative *i.e.* Δ*W* = – 20 *J*.

Problem **2.** An ideal gas is taken around *ABCA* as shown in the above *P*-*V* diagram. The work done during a cycle is

[KCET (Engg./Med.) 2001]



(a) 2*PV*

(b) *PV*

(c) 1/2*PV*

(d) Zero

Solution : (a) Work done = Area enclosed by triangle $ABC = \frac{1}{2}AC\times BC = \frac{1}{2}\times(3V-V)\times(3P-P) = 2\,PV$

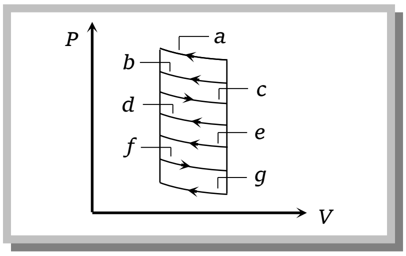
Problem 3. The *P-V* diagram shows seven curved paths (connected by vertical paths) that can be followed by a gas. Which two of them should be parts of a closed cycle if the net work done by the gas is to be at its maximum value

(a) *ac*

(b) *cg*

(c) *af*

(d) *cd*



Solution : (c) Area enclosed between *a* and *f* is maximum. So work done in closed cycles follows *a* and *f* is maximum.

Problem 4. If $C_v = 4.96\ cal/mole\ K$, then increase in internal energy when temperature of 2 moles of this gas is increased from 340 *K* to 342 *K*

(a) 27.80 *cal* (b) 19.84 *cal* (c) 13.90 *cal* (d) 9.92 *cal*

Solution : (b) Increase in internal energy $\Delta U = \mu.C_v.\Delta T = 2 \times 4.96 \times (342 - 340) = 2 \times 4.96 \times 2 = 19.84\ cal$

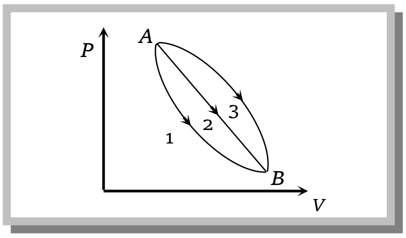
Problem 5. An ideal gas of mass *m* in a state *A* goes to another state *B* via three different processes as shown in figure. If Q_1, Q_2 and Q_3 denote the heat absorbed by the gas along the three paths, then **[MP PET 1992]**

(a) $Q_1 < Q_2 < Q_3$

(b) $Q_1 < Q_2 = Q_3$

(c) $Q_1 = Q_2 > Q_3$

(d) $Q_1 > Q_2 > Q_3$



Solution : (a) Area enclosed by curve 1 < Area enclosed by curve 2 < Area enclosed by curve 3

$\therefore$ $Q_1 < Q_2 < Q_3$ [As ΔU is same for all curves]

Problem 6. The relation between the internal energy *U* and adiabatic constant γ is

(a) $U = \frac{PV}{\gamma - 1}$ (b) $U = \frac{PV^{\gamma}}{\gamma - 1}$ (c) $U = \frac{PV}{\gamma}$ (d) $U = \frac{\gamma}{PV}$

Solution : (a) Change in internal energy = $\Delta U = \mu c_v \Delta T \Rightarrow U_2 - U_1 = \mu c_v (T_2 - T_1)$

Let initially $T_1 = 0$ so $U_1 = 0$ and finally $T_2 = T$ and $U_2 = U$

$$U = \mu\, c_v T = \mu T \times c_v = \frac{PV}{R} \times \frac{R}{\gamma - 1} = \frac{PV}{\gamma - 1} \quad \left[\text{As } PV = \mu RT \ \therefore\ \mu T = \frac{PV}{R} \text{ and } c_v = \frac{R}{\gamma - 1}\right]$$

13.4 Joule's Law

Whenever heat is converted into mechanical work or mechanical work is converted into heat, then the ratio of work done to heat produced always remains constant.

i.e. $W \propto Q$ or $\frac{W}{Q} = J$

This is Joule's law and *J* is called mechanical equivalent of heat.

Important points

(1) From $W = JQ$ if $Q = 1$ then $J = W$. Hence the amount of work done necessary to produce unit amount of heat is defined as the mechanical equivalent of heat.

(2) J is neither a constant, nor a physical quantity rather it is a conversion factor which used to convert *Joule* or *erg* into *calorie* or *kilo calories* vice-versa.

(3) Value of $J = 4.2 \dfrac{J}{calorie} = 4.2 \times 10^{7} \dfrac{erg}{calorie} = 4.2 \times 10^{3} \dfrac{J}{kilocalorie}$.

(4) When water in a stream falls from height h, then its potential energy is converted into heat and temperature of water rises slightly.

From $W = JQ$

$mgh = J\, ms\, \Delta t$ [where m = Mass, s = Specific heat of water]

$\therefore$ Rise in temperature $\Delta t = \dfrac{gh}{Js} {}^\circ C$

(5) The kinetic energy of a bullet fired from a gun gets converted into heat on striking the target. By this heat the temperature of bullet increases by Δt.

From $W = JQ$

$\dfrac{1}{2} mv^2 = J\, m\, s\, \Delta t$ [where m = Mass, v = Velocity of the bullet, s = Specific heat of the bullet]

$\therefore$ Rise in temperature $\Delta t = \dfrac{v^2}{2Js} {}^\circ C$

If the temperature of bullet rises upto the melting point of the bullet and bullet melts then.

From $W = JQ$

$\dfrac{1}{2} mv^2 = J[ms\, \Delta t + mL]$ [where L = Latent heat of bullet]

$\therefore$ Rise in temperature $\Delta t = \left\{ \left[\dfrac{v^2}{2J} - L \right] \Big/ s \right\} {}^\circ C$

(6) If ice-block falls down through some height and melts partially then its potential energy gets converted into heat of melting.

From $W = JQ$

$mgh = J\, m' L$ [where m = mass of ice block, m^1 = mass which melts]

So $m' = \dfrac{mgh}{JL} kg$

If ice-block completely melts down then $mgh = J\, mL$

$\therefore$ Height required for complete melting $h = \frac{JL}{g} meter$

Sample problems based on Joule's law

Problem 7. Water falls from a height of 210 m. Assuming whole of energy due to fall is converted into heat the rise in temperature of water would be (J = 4.3 *Joule/cal*)

(a) 42°C (b) 49°C (c) 0.49°C (d) 4.9°C

Solution : (c) Loss in potential energy of water = Increment in thermal energy of water

$\Rightarrow mgh = J \times ms\Delta t \Rightarrow 9.8 \times 210 = 4.3 \times 1000\ \Delta t \quad \therefore \quad \Delta t = 0.49°C$

Problem 8. A block of mass 100 gm slides on a rough horizontal surface. If the speed of the block decreases from 10 m/s to 5 m/s, the thermal energy developed in the process is

(a) 3.75 J (b) 37.5 J (c) 0.375 J (d) 0.75 J

Solution : (a) Thermal energy developed = Loss in kinetic energy $= \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2} \times 0.1 \times (10^2 - 5^2) = 3.75\ J$

Problem 9. The weight of a person is 60 kg. If he gets 10^5 *calories* heat through food and the efficiency of his body is 28%, then upto how much height he can climb (approximately)

(a) 100 m (b) 200 m (c) 400 m (d) 1000 m

Solution : (b) Increment in potential energy of mass = 28% of heat gained

$$\Rightarrow mgh = J\left(\frac{28}{100} \times 10^5\right) \Rightarrow 60 \times 9.8 \times h = 4.2 \times \left(\frac{28}{100} \times 10^5\right) \Rightarrow h = 200\ m$$

Problem 10. Hailstone at 0°C falls from a height of 1 km on an insulating surface converting whole of its kinetic energy into heat. What part of it will melt $(g = 10\,m/s^2)$

(a) $\frac{1}{33}$ (b) $\frac{1}{8}$ (c) $\frac{1}{33} \times 10^{-4}$ (d) All of it will melt

Solution : (a) Energy required for melting = Loss in potential energy $\Rightarrow$ $J \times m'L = mgh$

$\Rightarrow 4.18 \times (m' \times 80 \times 10^{-3}) = m \times 10 \times 1000 \Rightarrow \frac{m'}{m} = \frac{1}{33}$ [As $L = 80 \times 10^{-3}$ *calorie/kg*]

Problem 11. A bullet moving with a uniform velocity v, stops suddenly after hitting the target and the whole mass melts be m, specific heat S, initial temperature 25°C, melting point 475°C and the latent heat L. Then

(a) $mL = ms(475 - 25) + \frac{mv^2}{2J}$ (b) $ms(475 - 25) + mL = \frac{mv^2}{2J}$

(c) $ms(475 - 25) + mL = \frac{mv^2}{J}$ (d) $ms(475 - 25) - mL = \frac{mv^2}{2J}$

Solution : (b) K.E. of bullet = Heat required to raise the temperature of bullet from 25°C to 475°C + heat required to melt the bullet

$$\Rightarrow \frac{1}{2}mv^2 = J[ms(475 - 25) + mL] \Rightarrow ms(475 - 25) + mL = \frac{mv^2}{2J}.$$

Problem **12.** A lead bullet at 27°C just melts when stopped by an obstacle. Assuming that 25% of heat is absorbed by the obstacle, then the velocity of the bullet at the time of striking (M.P. of lead = 327°C, specific heat of lead = 0.03 $cal/gm°C$, latent heat of fusion of lead = 6 cal/gm and J = 4.2 J/cal) **[IIT-JEE 1981]**

(a) 410 m/sec (b) 1230 m/sec (c) 307.5 m/sec (d) None of these

Solution : (a) Using expression obtained in problem (11) we get $75\%\left(\frac{1}{2}mv^2\right) = J[ms(327-27+mL]$

Substituting $s = 0.3\times 10^3\ cal/kg^oC$ and $L = 6\times 10^3\ cal/kg$ we get $v = 410\ m/s$

Problem **13.** A drilling machine of power P watts is used to drill a hole in Cu block of mass m kg. If the specific heat of Cu is $5\ J\,kg^{-1}°C^{-1}$ and 40% of power lost due to heating of machine the rise in temperature of the block in T sec (will be in °C)

(a) $\frac{0.6\,PT}{ms}$ (b) $\frac{0.6\,P}{msT}$ (c) $\frac{0.4\,PT}{ms}$ (d) $\frac{0.4\,P}{msT}$

Solution : (a) As we know $\text{Power }(P) = \frac{\text{Work }(W)}{\text{Time }(T)}$ $\therefore W = P \times T$

As 40% energy is lost due to heating of machine so only 60% energy will increase the temperature of the block

$\therefore$ 60% of $W = m \times s \times \Delta t \Rightarrow \Delta t = \frac{0.6\,W}{ms} = \frac{0.6PT}{ms}$.

13.5 First Law of Thermodynamics

It is a statement of conservation of energy in thermodynamical process.

According to it heat given to a system (ΔQ) is equal to the sum of increase in its internal energy (ΔU) and the work done (ΔW) by the system against the surroundings.

$$\Delta Q = \Delta U + \Delta W$$

Important points

(1) It makes no distinction between work and heat as according to it the internal energy (and hence temperature) of a system may be increased either by adding heat to it or doing work on it or both.

(2) ΔQ and ΔW are the path functions but ΔU is the point function.

(3) In the above equation all three quantities ΔQ, ΔU and ΔW must be expressed either in *Joule* or in *calorie*.

(4) Just as zeroth law of thermodynamics introduces the concept of temperature, the first law introduces the concept of internal energy.

(5) Sign conventions

ΔQ	Positive	When heat is supplied to a system
	Negative	When heat is drawn from the system

ΔW	Positive	When work done by the gas (expansion)
	Negative	When work done on the gas (compression)
ΔU	Positive	When temperature increases, internal energy increases
	Negative	When temperature decreases, internal energy decreases

(6) When a thermos bottle is vigorously shaken :

No heat is transferred to the coffee $\Delta Q = 0$ [As thermos flask is insulated from the surrounding]

Work is done on the coffee against viscous force $\Delta W = (-)$

Internal energy of the coffee increases $\Delta U = (+)$

and temperature of the coffee also increases $\Delta T = (+)$

(7) Limitation : First law of thermodynamics does not indicate the direction of heat transfer. It does not tell anything about the conditions, under which heat can be transformed into work and also it does not indicate as to why the whole of heat energy cannot be converted into mechanical work continuously.

Sample problems based on First law of thermodynamics

***Problem* 14.** If 150 *J* of heat is added to a system and the work done by the system is 110 *J*, then change in internal energy will be

(a) 260 *J* (b) 150 *J* (c) 110 *J* (d) 40 *J*

Solution : (d) $\Delta Q = \Delta U + \Delta W \Rightarrow 150 = \Delta U + 110 \Rightarrow \Delta U = 40\ J$

***Problem* 15.** 110 *J* of heat is added to a gaseous system, whose internal energy change is 40 *J*, then the amount of external work done is

(a) 150 *J* (b) 70 *J* (c) 110 *J* (d) 40 *J*

Solution : (b) $\Delta Q = \Delta U + \Delta W \Rightarrow 110 = 40 + \Delta W \Rightarrow \Delta W = 70\ J$

***Problem* 16.** When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas, is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

Solution : (d) $\frac{\Delta U}{\Delta Q} = \frac{C_v \Delta T}{C_P \Delta T} = \frac{R/(\gamma-1)}{\gamma R/(\gamma-1)} = \frac{1}{\gamma} = \frac{1}{7/5} = \frac{5}{7}$

***Problem* 17.** An electric fan is switched on in a closed room. The air in the room is

(a) Cooled

(b) Heated

(c) Maintains its temperature

(d) Heated or cooled depending on the atmospheric pressure

Solution : (b) When an electric fan is switched on in a closed room conventional current of air flows. Hence due to viscous force mechanical energy is converted into heat and some heat is also produced due to thermal effect of electric current in motor of fan.

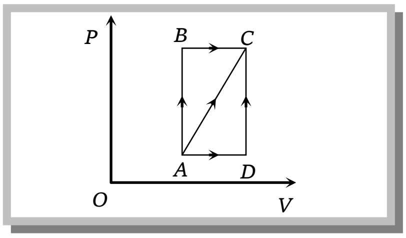
Problem **18.** A gas is compressed at a constant pressure of $50\ N/m^2$ from a volume of $10\ m^3$ to a volume of $4\ m^3$. Energy of 100 *J* is then added to the gas by heating. Its internal energy is

(a) Increased by 400 *J* (b) Increased by 200 *J* (c) Increased by 100 *J* (d)Decreased by 200 *J*

Solution : (a) $\Delta Q = \Delta U + \Delta W \Rightarrow \Delta Q = \Delta U + \Delta dV \Rightarrow 100 = \Delta U + 50(4-10) \Rightarrow 100 = \Delta U - 300 \quad \therefore \Delta U = 400\ J$

Problem **19.** A thermodynamic process is shown in the figure. The pressures and volumes corresponding to some points in the figure are : $P_A = 3\times 10^4\,Pa, P_B = 8\times 10^4\,Pa$ and $V_A = 2\times 10^{-3}\,m^3$, $V_D = 5\times 10^{-3}\,m^3$

In process *AB*, 600 *J* of heat is added to the system and in process *BC*, 200 *J* of heat is added to the system. The change in internal energy of the system in process *AC* would be



(a) 560 *J*

(b) 800 *J*

(c) 600 *J*

(d) 640 *J*

Solution : (a) By adjoining graph $W_{AB} = 0$ and $W_{BC} = 8\times 10^4[5-2]\times 10^{-3} = 240\ J$

$\therefore\ W_{AC} = W_{AB} + W_{BC} = 0 + 240 = 240\ J$

Now, $\Delta Q_{AC} = \Delta Q_{AB} + \Delta Q_{BC} = 600 + 200 = 800\ J$

From first law of thermodynamics $\Delta Q_{AC} = \Delta U_{AC} + \Delta W_{AC} \Rightarrow 800 = \Delta U_{AC} + 240 \Rightarrow \Delta U_{AC} = 560\ J.$

Problem **20.** If *R* = universal gas constant, the amount of heat needed to raise the temperature of 2 *mole* of an ideal monoatomic gas from 273 *K* to 373 *K* when no work is done

(a) 100 *R* (b) 150 *R* (c) 300 *R* (d) 500 *R*

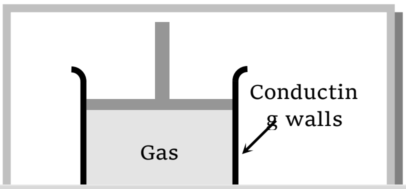
Solution : (c) $\Delta Q = \Delta U = \mu C_v \Delta T = \mu\left(\frac{R}{\gamma-1}\right)\Delta T = 2\times\frac{R}{\frac{5}{3}-1}[373-273] = 300\ R$ [As for monoatomic gas $\gamma = \frac{5}{3}$]

13.6 Isothermal Process

When a thermodynamic system undergoes a physical change in such a way that its temperature remains constant, then the change is known as isothermal changes.

In this process, *P* and *V* change but *T* = constant *i.e.* change in temperature $\Delta T = 0$

(1) **Essential condition for isothermal process**



(i) The walls of the container must be perfectly conducting to allow free exchange of heat between the gas and its surrounding.

(ii) The process of compression or expansion should be so slow so as to provide time for the exchange of heat.

Since these two conditions are not fully realised in practice, therefore, no process is perfectly isothermal.

(2) **Equation of state :** From ideal gas equation $PV = \mu RT$

If temperature remains constant then PV= constant *i.e.* in all isothermal process Boyle's law is obeyed.

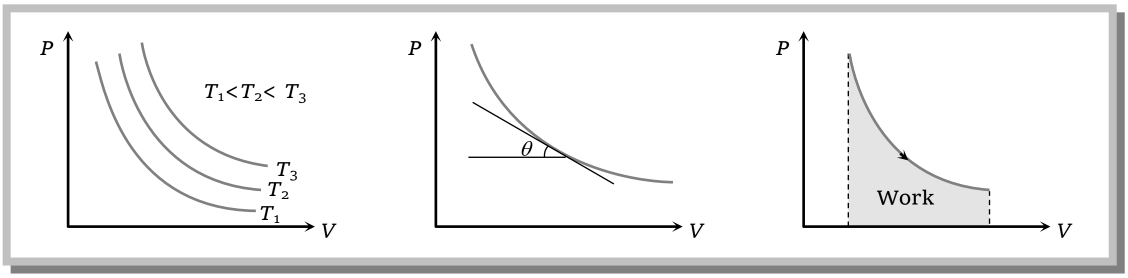
Hence equation of state is PV = constant.

(3) **Example of isothermal process**

(i) Melting process [Ice melts at constant temperature 0°C]

(ii) Boiling process [water boils at constant temperature 100°C].

(4) **Indicator diagram**



(i) Curves obtained on PV graph are called isotherms and they are hyperbolic in nature.

(ii) Slope of isothermal curve : By differentiating PV = constant. We get

$$P\,dV + V\,dP = 0 \quad \Rightarrow P\,dV = -V\,dP \quad \Rightarrow \frac{dP}{dV} = -\frac{P}{V}$$

$$\therefore \quad \tan\theta = \frac{dP}{dV} = -\frac{P}{V}$$

(iii) Area between the isotherm and volume axis represents the work done in isothermal process.

If volume increases ΔW = + Area under curve and if volume decreases ΔW = – Area under curve

(5) **Specific heat :** Specific heat of gas during isothermal change is infinite.

As $$C = \frac{Q}{m\Delta T} = \frac{Q}{m \times 0} = \infty \qquad [\text{As } \Delta T = 0]$$

(6) **Isothermal elasticity :** For isothermal process PV = constant

Differentiating both sides $PdV + VdP = 0 \Rightarrow P\,dV = -V\,dP \Rightarrow P = \dfrac{dP}{-dV/V} = \dfrac{\text{Stress}}{\text{Strain}} = E_\theta$

$\therefore \quad E_\theta = P$ *i.e.* isothermal elasticity is equal to pressure

At N.T.P. isothermal elasticity of gas = Atmospheric pressure = $1.01 \times 10^5\ N/m^2$

(7) **Work done in isothermal process**

$$W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{\mu RT}{V} dV \qquad [\text{As } PV = \mu RT]$$

$$W = \mu RT \log_e\left(\frac{V_f}{V_i}\right) = 2.303\ \mu RT\ \log_{10}\left(\frac{V_f}{V_i}\right)$$

or
$$W = \mu RT \log_e\left(\frac{P_i}{P_f}\right) = 2.303\,\mu RT\ \log_{10}\left(\frac{P_i}{P_f}\right)$$

(8) **FLTD in isothermal process**

$\Delta Q = \Delta U + \Delta W$ but $\Delta U \propto \Delta T$

$\therefore \quad \Delta U = 0 \qquad [\text{As } \Delta T = 0]$

$\therefore \quad \Delta Q = \Delta W$ *i.e.* heat supplied in an isothermal change is used to do work against external surrounding.

or if the work is done on the system than equal amount of heat energy will be liberated by the system.

Sample problems based on Isothermal process

***Problem* 21.** One mole of O_2 gas having a volume equal to 22.4 *litres* at 0°*C* and 1 atmospheric pressure in compressed isothermally so that its volume reduces to 11.2 *litres*. The work done in this process is **[MP PET 1993; BVP 2003]**

(a) 1672.5 *J* (b) 1728 *J* (c) – 1728 *J* (d) – 1572.5 *J*

Solution : (d) Work done in an adiabatic process

$$W = \mu RT \log_e\left(\frac{V_f}{V_i}\right) = 1 \times 8.3 \times 273 \times \log_e\left(\frac{11.2}{22.4}\right) = 8.3 \times 273 \times (-0.69) \approx -1572\ J$$

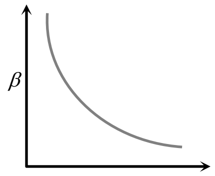
***Problem* 22.** An ideal gas *A* and a real gas *B* have their volumes increased from *V* to 2*V* under isothermal conditions. The increase in internal energy

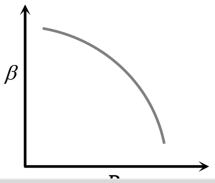
(a) Will be same in both *A* and *B* (b) Will be zero in both the gases

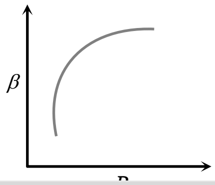
(c) Of *B* will be more than that of *A* (d) Of *A* will be more than that of *B*

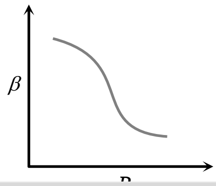
Solution : (c) In real gases an additional work is also done in expansion due to intermolecular attraction.

***Problem* 23.** Which of the following graphs correctly represents the variation of $\beta = -(dV/dP)/V$ with *P* for an ideal gas at constant temperature

(a)


(b)


(c)


(d)


Solution : (a) For an isothermal process PV = constant $\Rightarrow$ $PdV + VdP = 0 \Rightarrow -\frac{1}{V}\left(\frac{dV}{dP}\right) = \frac{1}{P}$

So, $\beta = \frac{1}{P}$ $\therefore$ graph will be rectangular hyperbola.

Problem **24.** Consider the following statements

Assertion (A): The internal energy of an ideal gas does not change during an isothermal process

Reason (R) : The decrease in volume of a gas is compensated by a corresponding increase in pressure when its temperature is held constant.

Of these statements **[SCRA 1994]**

(a) Both A and R are true and R is a correct explanation of A

(b) Both A and R are true but R is not a correct explanation of A

(c) A is true but R is false

(d) Both A and R are false

(e) A is false but R is true

Solution : (b) As $\Delta U \propto \Delta T$ so for isothermal process internal energy will not change and also $P \propto \frac{1}{V}$ (Boyle's law)

Problem **25.** How much energy is absorbed by 10 *kg* molecule of an ideal gas if it expands from an initial pressure of 8 *atm* to 4 *atm* at a constant temperature of 27°*C*

(a) $1.728 \times 10^7 J$ (b) $17.28 \times 10^7 J$ (c) $1.728 \times 10^9 J$ (d) $17.28 \times 10^9 J$

Solution : (a) Work done in an isothermal process

$$W = \mu RT \log_e\left(\frac{P_i}{P_f}\right) = (10 \times 10^3) \times 8.3 \times 300 \times \log_e\left(\frac{8}{4}\right) = 10^4 \times 8.3 \times 300 \times 0.693 = 1.728 \times 10^7 \ J$$

Problem **26.** 5 *moles* of an ideal gas undergoes an isothermal process at 500*K* in which its volume is doubled. The work done by the gas system is

(a) 3500 J (b) 14400 J (c) 17800 J (d) 5200 J

Solution : (b) $\Delta W = \mu RT \log_e\left(\frac{V_f}{V_i}\right) = 5 \times 8.3 \times 500 \times \log_e\left(\frac{2V}{V}\right) = 5 \times 8.3 \times 500 \times 0.69 \approx 14400 \ J.$

Problem **27.** Work done by a system under isothermal change from a volume V_1 to V_2 for a gas which obeys Vander Waal's equation $(V - \beta n)\left(P + \frac{\alpha n^2}{V}\right) = nRT$

(a) $nRT \log_e\left(\frac{V_2 - n\beta}{V_1 - n\beta}\right) + \alpha n^2\left(\frac{V_1 - V_2}{V_1 V_2}\right)$ (b) $nRT \log_{10}\left(\frac{V_2 - \alpha\beta}{V_1 - \alpha\beta}\right) + \alpha n^2\left(\frac{V_1 - V_2}{V_1 V_2}\right)$

(c) $nRT \log_e\left(\frac{V_2 - n\alpha}{V_1 - n\alpha}\right) + \beta n^2\left(\frac{V_1 - V_2}{V_1 V_2}\right)$ (d) $nRT \log_e\left(\frac{V_1 - n\beta}{V_2 - n\beta}\right) + \alpha n^2\left(\frac{V_1 V_2}{V_1 - V_2}\right)$

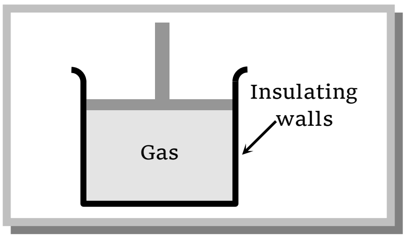
Solution : (a) By Vander Waal's equation $P = \frac{nRT}{V - n\beta} - \frac{\alpha n^2}{V^2}$

Work done, $W = \int_{V_1}^{V_2} P dV = nRT \int_{V_1}^{V_2} \frac{dV}{V - n\beta} - \alpha n^2 \int_{V_1}^{V_2} \frac{dV}{V^2}$

$$= nRT\left[\log_e(V - n\beta)\right]_{V_1}^{V_2} + \alpha n^2\left[\frac{1}{V}\right]_{V_1}^{V_2} = nRT \log_e \frac{V_2 - n\beta}{V_1 - n\beta} + \alpha n^2\left(\frac{V_1 - V_2}{V_1 V_2}\right)$$

13.7 Adiabatic Process

When a thermodynamic system undergoes a change in such a way that no exchange of heat takes place between it and the surroundings, the process is known as adiabatic process.



In this process P, V and T changes but ΔQ = 0.

(1) **Essential conditions for adiabatic process**

(i) There should not be any exchange of heat between the system and its surroundings. All walls of the container and the piston must be perfectly insulating.

(ii) The system should be compressed or allowed to expand suddenly so that there is no time for the exchange of heat between the system and its surroundings.

Since, these two conditions are not fully realised in practice, so no process is perfectly adiabatic.

(2) **Example of some adiabatic process**

(i) Sudden compression or expansion of a gas in a container with perfectly non-conducting walls.

(ii) Sudden bursting of the tube of bicycle tyre.

(iii) Propagation of sound waves in air and other gases.

(iv) Expansion of steam in the cylinder of steam engine.

(3) **FLTD in adiabatic process :** $\Delta Q = \Delta U + \Delta W$

but for adiabatic process $\Delta Q = 0 \;\; \therefore \;\; \Delta U + \Delta W = 0$

If ΔW = positive then ΔU = negative so temperature decreases *i.e.* adiabatic expansion produce cooling.

If ΔW = negative then ΔU = positive so temperature increases *i.e.* adiabatic compression produce heating.

(4) **Equation of state :** As in case of adiabatic change first law of thermodynamics reduces to,

$$\Delta U + \Delta W = 0\,, \qquad i.e.,\; dU + dW = 0 \qquad \text{..... (i)}$$

But as for an ideal gas $dU = \mu C_V dT$ and $dW = PdV$

Equation (i) becomes $\mu C_v dT + P\,dV = 0$ ..... (ii)

But for a gas as $PV = \mu RT$, $P\,dV + V\,dP = \mu R\,dT$ ..... (iii)

So eliminating dT between equation (ii) and (iii) $\mu C_v \dfrac{(P\,dV + V\,dP)}{\mu R} + P\,dV = 0$

or $$\frac{(P\,dV + V\,dP)}{(\gamma - 1)} + P\,dV = 0 \qquad \left[\text{as } C_v = \frac{R}{(\gamma-1)}\right]$$

or $$\gamma P\,dV + V\,dP = 0 \quad \textit{i.e.,} \quad \gamma\frac{dV}{V} + \frac{dP}{P} = 0$$

Which on integration gives

$$\gamma \log_e V + \log_e P = C\,,$$

i.e., $$\log(PV^{\gamma}) = C$$

or $$PV^{\gamma} = \text{constant} \qquad \text{..... (iv)}$$

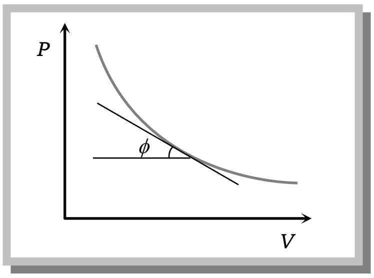
Equation (iv) is called equation of state for adiabatic change and can also be re-written as

$$TV^{\gamma-1} = \text{constant} \qquad [\text{as } P = (\mu RT/V)] \qquad \text{..... (iv)}$$

and $$\frac{T^{\gamma}}{P^{\gamma-1}} = \text{constant} \qquad \left[\text{as } V = \frac{\mu RT}{P}\right] \qquad \text{..... (vi)}$$

(5) **Indicator diagram**

(i) Curve obtained on PV graph are called adiabatic curve.



(ii) Slope of adiabatic curve : From $PV^{\gamma} = \text{constant}$

By differentiating, we get $dP\,V^{\gamma} + P\gamma V^{\gamma-1}\,dV = 0$

$$\frac{dP}{dV} = -\gamma\frac{PV^{\gamma-1}}{V^{\gamma}} = -\gamma\left(\frac{P}{V}\right)$$

$\therefore$ Slope of adiabatic curve $\tan\phi = -\gamma\left(\dfrac{P}{V}\right)$

But we also know that slope of isothermal curve $\tan\phi = \dfrac{-P}{V}$

So, $$\frac{\text{Slope of adiabatic curve}}{\text{Slope of isothermal curve}} = \frac{-\gamma(P/V)}{-(P/V)} = \gamma = \frac{C_p}{C_v} > 1$$

(6) **Specific heat :** Specific heat of a gas during adiabatic change is zero

As $$C = \frac{Q}{m\Delta T} = \frac{0}{m\Delta T} = 0 \qquad [\text{As } Q = 0]$$

(7) **Adiabatic elasticity :** For adiabatic process $PV^{\gamma} = \text{constant}$

Differentiating both sides $d\,PV^{\gamma} + P\gamma V^{\gamma-1}dV = 0$

$$\gamma P = \frac{dP}{-dV/V} = \frac{\text{Stress}}{\text{Strain}} = E_{\phi}$$

$$E_{\phi} = \gamma P$$

i.e. adiabatic elasticity is γ times that of pressure but we know isothermal elasticity $E_{\theta} = P$

So $$\frac{E_{\phi}}{E_{\theta}} = \frac{\text{Adiabatic elasticity}}{\text{Isothermal elasticity}} = \frac{\gamma P}{P} = \gamma$$

i.e. the ratio of two elasticities of gases is equal to the ratio of two specific heats.

(8) **Work done in adiabatic process**

$$W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{K}{V^{\gamma}}dV \qquad \left[\text{As}\left(P = \frac{K}{V^{\gamma}}\right)\right]$$

or $$= \frac{1}{[1-\gamma]}\left[\frac{K}{V_f^{\gamma-1}} - \frac{K}{V_i^{\gamma-1}}\right] \qquad \left[\text{As}\int V^{-\gamma}dV = \frac{V^{-\gamma+1}}{(-\gamma+1)}\right]$$

or $$= \frac{[P_f V_f - P_i V_i]}{(1-\gamma)} \qquad [\text{As}\, K = PV^{\gamma} = P_f V_f^{\gamma} = P_i V_i^{\gamma}]$$

or $$= \frac{\mu R}{(1-\gamma)}[T_f - T_i] \qquad [\text{As } P_f V_f = \mu R T_f \text{ and } P_i V_i = \mu R T_i\,]$$

So $$= \frac{[P_i V_i - P_f V_f]}{(\gamma-1)} = \frac{\mu R(T_i - T_f)}{(\gamma-1)}$$

(9) **Free expansion :** Free expansion is adiabatic process in which no work is performed on or by the system. Consider two vessels placed in a system which is enclosed with thermal insulation (asbestos-covered). One vessel contains a gas and the other is evacuated. The two vessels are connected by a stopcock. When suddenly the stopcock is opened, the gas rushes into the evacuated vessel and expands freely. The process is adiabatic as the vessels are placed in thermal insulating system ($dQ = 0$) moreover, the walls of the vessel are rigid and hence no external work is performed ($dW = 0$).

Now according to the first law of thermodynamics $dU = 0$

If U_i and U_f be the initial and final internal energies of the gas

then $U_f - U_i = 0$ $\qquad$ $[\text{As } U_f = U_i\,]$

Thus the final and initial energies are equal in free expansion.

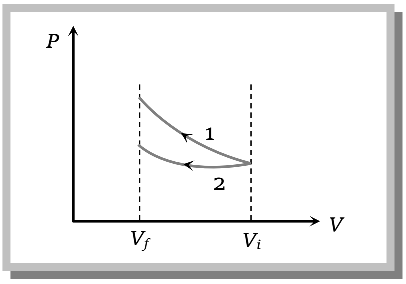
(10) **Special cases of adiabatic process**

$$PV^{\gamma} = \text{constant} \quad \therefore\ P \propto \frac{1}{V^{\gamma}}, \quad PT^{\frac{\gamma}{1-\gamma}} = \text{constant} \quad \therefore\ P \propto T^{\frac{\gamma}{\gamma-1}} \quad \text{and} \quad TV^{\gamma-1} = \text{constant} \quad \therefore\ T \propto \frac{1}{V^{\gamma-1}}$$

Type of gas	$P \propto \frac{1}{V^{\gamma}}$	$P \propto T^{\frac{\gamma}{\gamma-1}}$	$T \propto \frac{1}{V^{\gamma-1}}$
Monoatomic $\gamma = 5/3$	$P \propto \frac{1}{V^{5/3}}$	$P \propto T^{5/2}$	$T \propto \frac{1}{V^{2/3}}$
Diatomic $\gamma = 7/5$	$P \propto \frac{1}{V^{7/5}}$	$P \propto T^{7/2}$	$T \propto \frac{1}{V^{2/5}}$
Polyatomic $\gamma = 4/3$	$P \propto \frac{1}{V^{4/3}}$	$P \propto T^{4}$	$T \propto \frac{1}{V^{1/3}}$

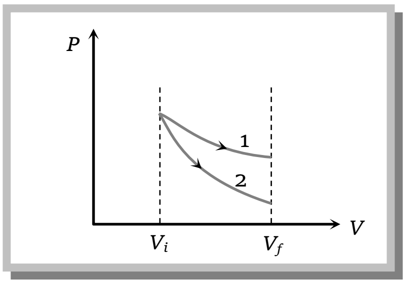
(11) Comparison between isothermal and adiabatic process

(i) **Compression :** If a gas is compressed isothermally and adiabatically from volume V_i to V_f then from the slope of the graph it is clear that graph 1 represents adiabatic process where as graph 2 represent isothermal process.



Work done	$W_{\text{adiabatic}} > W_{\text{isothermal}}$
Final pressure	$P_{\text{adiabatic}} > P_{\text{isothermal}}$
Final temperature	$T_{\text{adiabatic}} > T_{\text{isothermal}}$

(ii) **Expansion :** If a gas expands isothermally and adiabatically from volume V_i to V_f then from the slope of the graph it is clear that graph 1 represent isothermal process, graph 2 represent adiabatic process.



Work done	$W_{\text{isothermal}} > W_{\text{adiabatic}}$
Final pressure	$P_{\text{isothermal}} > P_{\text{adiabatic}}$
Final temperature	$T_{\text{isothermal}} > T_{\text{adiabatic}}$

Sample problems based on Adiabatic process

Problem **28.** During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_p / C_v for the gas is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$ (c) 2 (d) $\frac{5}{3}$

Solution : (a) Given $P \propto T^3$. But for adiabatic process $P \propto T^{\frac{\gamma}{\gamma-1}}$. So, $\frac{\gamma}{\gamma-1} = 3 \Rightarrow \gamma = \frac{3}{2} \Rightarrow \frac{C_p}{C_v} = \frac{3}{2}$

Problem **29.** An ideal gas at 27°C is compressed adiabatically to $\frac{8}{27}$ of its original volume. If $\gamma = \frac{5}{3}$, then the rise in temperature is

(a) 450 K (b) 375 K (c) 225 K (d) 405 K

Solution : (b) For an adiabatic process $TV^{\gamma-1}$ = constant

$$\therefore \frac{T_1}{T_2}=\left[\frac{V_2}{V_1}\right]^{\gamma-1} \Rightarrow T_2=T_1\left[\frac{V_1}{V_2}\right]^{\gamma-1}=300\left[\frac{27}{8}\right]^{\frac{5}{3}-1}=300\left[\frac{27}{8}\right]^{2/3}=675\ K \Rightarrow \Delta T=675-300=375\ K$$

Problem **30.** If γ = 2.5 and volume is equal to 1/8 times to the initial volume then pressure P' is equal to (initial pressure = P)

[RPET 2003]

(a) $P'=P$ (b) $P'=2P$ (c) $P'=P\times(2)^{15/2}$ (d) $P'=7P$

Solution : (c) For an adiabatic process PV^{γ} = constant $\Rightarrow \frac{P_2}{P_1}=\left[\frac{V_1}{V_2}\right]^{\gamma} \Rightarrow \frac{P'}{P}=8^{5/2} \Rightarrow P'=P\times(2)^{15/2}$

Problem **31.** In adiabatic expansion of a gas

(a) Its pressure increases (b) Its temperature falls

(c) Its density decreases (d) Its thermal energy increases

Solution : (b) $\Delta Q=\Delta U+\Delta W$, In an adiabatic process ΔQ = 0 $\therefore -\Delta U=\Delta W$

In expansion ΔW = positive $\therefore$ ΔU = negative. Hence internal energy *i.e.* temperature decreases.

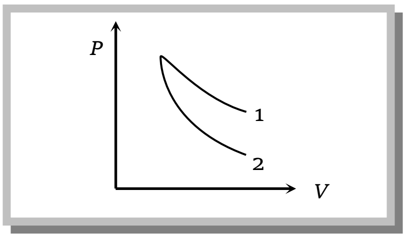
Problem **32.** *P-V* plots for two gases during adiabatic process are shown in the figure. Plots 1 and 2 should correspond respectively to

(a) *He* and O_2

(b) O_2 and *He*

(c) *He* and *Ar*

(d) O_2 and N_2



Solution : (b) Slope of adiabatic curve $\propto \gamma \propto \frac{1}{\text{Atomicity of the gas}}$. So γ is inversely proportional to atomicity of the gas.

[As $\gamma=1.66$ for monoatomic gas, γ = 1.4 for diatomic gas and γ = 1.33 for triatomic non-linear gas.]

From the graph it is clear that slope of the curve 1 is less so this should be adiabatic curve for diatomic gas (*i.e.* O_2).

Similarly slope of the curve 2 is more so it should be adiabatic curve for monoatomic gas (*i.e.* *He*).

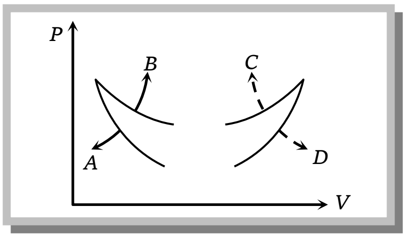
Problem **33.** A monoatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by

[IIT-JEE (Screening) 2000]

(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$ (b) $\frac{L_1}{L_2}$ (c) $\frac{L_2}{L_1}$ (d) $\left(\frac{L_2}{L_1}\right)^{2/3}$

Solution : (d) For an adiabatic process $T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1} \Rightarrow \frac{T_1}{T_2}=\left[\frac{V_2}{V_1}\right]^{\gamma-1}=\left[\frac{L_2A}{L_1A}\right]^{5/3-1}=\left[\frac{L_2}{L_1}\right]^{2/3}$.

Problem **34.** Four curves *A*, *B*, *C* and *D* are drawn in the adjoining figure for a given amount of gas. The curves which represent adiabatic and isothermal changes are



(a) *C* and *D* respectively

(b) *D* and *C* respectively

(c) *A* and *B* respectively

(d) *B* and *A* respectively

Solution : (c) As we know that slope of isothermal and adiabatic curves are always negative and slope of adiabatic curve is always greater than that of isothermal curve so is the given graph curve *A* and curve *B* represents adiabatic and isothermal changes respectively.

Problem **35.** A thermally insulated container is divided into two parts by a screen. In one part the pressure and temperature are *P* and *T* for an ideal gas filled. In the second part it is vacuum. If now a small hole is created in the screen, then the temperature of the gas will

(a) Decrease (b) Increase (c) Remain same (d) None of these

Solution : (c) In second part there is a vacuum *i.e.* $P = 0$. So work done in expansion $= P\Delta V = 0$

Problem **36.** Two samples *A* and *B* of a gas initially at the same pressure and temperature are compressed from volume *V* to *V*/2 (*A* isothermally and *B* adiabatically). The final pressure of *A* is **[MP PET 1996, 99; MP PMT 1997, 99]**

(a) Greater than the final pressure of *B* (b) Equal to the final pressure of *B*

(c) Less than the final pressure of *B* (d) Twice the final pressure of *B*

Solution : (c) For isothermal process $P_1V=P_2^{'}\frac{V}{2} \Rightarrow P_2^{'}=2P_1$ .....(i)

For adiabatic process $P_1V^{\gamma}=P_2\left(\frac{V}{2}\right)^{\gamma} \Rightarrow P_2=2^{\gamma}P_1$ .....(ii)

Since $\gamma>1$. Therefore $P_2>P_2^{'}$

Problem **37.** A gas has pressure *P* and volume *V*. It is now compressed adiabatically to $\frac{1}{32}$ times the original volume. If $(32)^{1.4}=128$, the final pressure is

(a) 32 *P* (b) 128 *P* (c) $\frac{P}{128}$ (d) $\frac{P}{32}$

Solution : (b) For an adiabatic process $P_1V_1^{\gamma}=P_2V_2^{\gamma} \Rightarrow \frac{P_2}{P_1}=\left(\frac{V_1}{V_2}\right)^{\gamma}=\left(\frac{V}{V/32}\right)^{1.4}=(32)^{1.4}=128$

$\therefore$ Final pressure = 128 P.

13.8 Isobaric Process

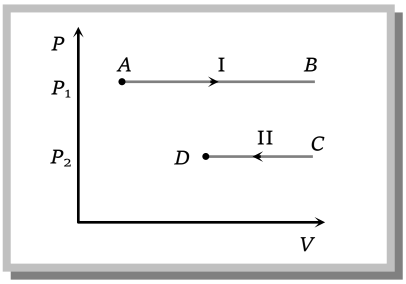
When a thermodynamic system undergoes a physical change in such a way that its pressure remains constant, then the change is known as isobaric process.

In this process V and T changes but P remains constant. Hence Charle's law is obeyed in this process.

(1) **Equation of state :** From ideal gas equation $PV = \mu RT$

If pressure remains constant $V \propto T$ or

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{constant}$$



(2) **Indicator diagram :** Graph I represent isobaric expansion, graph II represent isobaric compression.

Slope of indicator diagram $\frac{dP}{dV} = 0$

(3) **Specific heat :** Specific heat of gas during isobaric process $C_P = \left(\frac{f}{2} + 1\right)R$

(4) **Bulk modulus of elasticity :** $K = \frac{\Delta P}{-\Delta V / V} = 0$ [As ΔP = 0]

(5) **Work done in isobaric process :** $\Delta W = \int_{V_i}^{V_f} P\, dV = P\int_{V_i}^{V_f} dV = P[V_f - V_i]$ [As P = constant]

$$\therefore \quad \Delta W = P(V_f - V_i) = \mu R[T_f - T_i] = \mu R\, \Delta T$$

(6) **FLTD in isobaric process :** $\Delta U = \mu C_V\, \Delta T = \mu \frac{R}{(\gamma - 1)} \Delta T$ and $\Delta W = \mu R\, \Delta T$

From FLTD $\Delta Q = \Delta U + \Delta W$

$$\therefore \quad \Delta Q = \mu \frac{R}{(\gamma-1)} \Delta T + \mu R\, \Delta T = \mu R\, \Delta T \left[\frac{1}{\gamma - 1} + 1\right] = \mu R\, \Delta T \frac{\gamma}{\gamma - 1} = \mu \left(\frac{\gamma}{\gamma - 1}\right) R\, \Delta T$$

$$\Delta Q = \mu C_P\, \Delta T$$

(7) **Examples of isobaric process**

(i) Conversion of water into vapour phase (boiling process) :

When water gets converted into vapour phase, then its volume increases. Hence some part of absorbed heat is used up to increase the volume against external pressure and remaining amount of heat is used up to increase the internal potential energy of the molecules (because interatomic forces of attraction takes place between the molecules of system and when the distance between them increases, then its potential energy increases. It must be noted that during change of state since temperature remains constant, hence there will be no increase in internal kinetic energy of the molecules).

From first law of thermodynamics $\Delta Q = \Delta U + \Delta W = \Delta U_K + \Delta U_P + \Delta W = 0 + \Delta U_P + P\Delta V$

$\therefore \qquad \Delta U_P = \Delta Q - P[V_f - V_i]$

or $\qquad \Delta U_P = mL - P[V_f - V_i] \qquad$ [As $\Delta Q = mL$]

(ii) Conversion of ice into water

$$\Delta Q = \Delta U + \Delta W = \Delta U_P + \Delta U_K + \Delta W$$

$mL = \Delta U_P + 0 + P[V_f - V_i] \qquad$ [$\Delta U_K = 0$ as there is no change in temperature]

$\Delta U_P = mL \qquad$ [when ice convert into water then change in volume is negligible]

Note : ☐ In isobaric compression, temperature increases and internal energy flows out in the form of heat energy, while in isobaric expansion, temperature increases and heat flows into the system.

☐ Isobaric expansion of the volume of a gas is given by $V_t = V_0(1 + \gamma_V t)$

where $\gamma_V = \frac{1}{273}\, per°C =$ coefficient of volume expansion.

Sample problems based on Isobaric process

***Problem* 38.** $1\,cm^3$ of water at its boiling point absorbs 540 *calories* of heat to become steam with a volume of $1671\,cm^3$. If the atmospheric pressure is $1.013 \times 10^5\ N/m^2$ and the mechanical equivalent of heat = 4.19 *J/calorie*, the energy spent in this process in overcoming intermolecular forces is **[MP PET 1999, 2001; Orissa JEE 2002]**

(a) 540 *calorie* (b) 40 *calorie* (c) 500 *calorie* (d) Zero

Solution : (c) Energy spent in overcoming inter molecular forces $\Delta U = \Delta Q - \Delta W$

$$= \Delta Q - P(V_2 - V_1) = 540 - \frac{1.013 \times 10^5 (1671 - 1) \times 10^{-6}}{4.2} \approx 500\ calorie$$

***Problem* 39.** A gas expands $0.25\,m^3$ at constant pressure $10^3\ N/m^2$, the work done is

[CPMT 1997; UPSEAT 1999; JIPMER 2001, 2002]

(a) 2.5 *ergs* (b) 250 *J* (c) 250 *W* (d) 250 *N*

Solution : (b) As we know, work done $= P\Delta V = 10^3 \times 0.25 = 250\ J$

***Problem* 40.** 5 *mole* of hydrogen gas is heated from 30°*C* to 60°*C* at constant pressure. Heat given to the gas is (given *R* = 2 *cal/mole degree*)

(a) 750 *calorie* (b) 630 *calorie* (c) 1050 *calorie* (d) 1470 *calorie*

Solution : (c) $(\Delta Q)_p = \mu C_p \Delta T = \mu\left(\frac{\gamma}{\gamma - 1}\right) R\,\Delta T$

$$\therefore\ (\Delta Q)_p = 5\times\left(\frac{\frac{7}{5}}{\frac{7}{5}-1}\right)\times 2\times 30 = 5\times 2\times\frac{7}{5}\times\frac{5}{2}\times 30 = 1050\ \textit{calorie}\ \ [\text{As } \mu = 5\ \textit{mole} \text{ and } \gamma = \frac{7}{5} \text{ for } H_2]$$

Problem **41.** The latent heat of vaporisation of water is 2240 *J/gm*. If the work done in the process of expansion of 1*g* is 168 *J*, then increase in internal energy is

(a) 2408 *J* (b) 2240 *J* (c) 2072 *J* (d) 1904 *J*

Solution : (c) $\Delta Q = \Delta U + \Delta W \Rightarrow 2240 = \Delta U + 168 \Rightarrow \Delta U = 2072\ J$

Problem **42.** When an ideal gas (γ = 5/3) is heated under constant pressure, then what percentage of given heat energy will be utilised in doing external work

(a) 40% (b) 30% (c) 60% (d) 20%

Solution : (a) $\dfrac{\Delta W}{\Delta Q} = \dfrac{\Delta Q - \Delta U}{\Delta Q} = \dfrac{C_p - C_v}{C_p} = 1 - \dfrac{1}{\gamma} = 1 - \dfrac{1}{1-\frac{5}{3}} = \dfrac{2}{5}$

i.e. percentage energy utilised in doing external work = $\dfrac{2}{5}\times 100$ = 40%.

Problem **43.** At 100°*C* the volume of 1*kg* of water is $10^{-3}\ m^3$ and volume of 1 *kg* of steam at normal pressure is $1.671\ m^3$. The latent heat of steam is $2.3\times 10^6\ J/kg$ and the normal pressure is $10^5\ N/m^2$. If 5 *kg* of water at 100°*C* is converted into steam, the increase in the internal energy of water in this process will be

(a) $8.35\times 10^5 J$ (b) $10.66\times 10^6 J$ (c) $11.5\times 10^6 J$ (d) Zero

Solution : (b) Heat required to convert 5 *kg* of water into steam $\Delta Q = mL = 5\times 2.3\times 10^6 = 11.5\times 10^6\ J$

Work done in expanding volume, $\Delta W = P\Delta V = 5\times 10^5[1.671 - 10^{-3}] = 0.835\times 10^6\ J$

Now by first law of thermodynamics $\Delta U = \Delta Q - \Delta W \Rightarrow$ $\Delta U = 11.5\times 10^6 - 0.835\times 10^6 = 10.66\times 10^6\ J$

13.9 Isochoric or Isometric Process

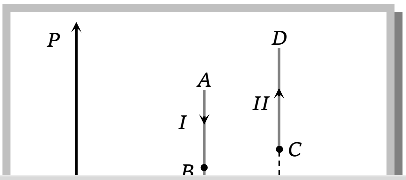
When a thermodynamic process undergoes a physical change in such a way that its volume remains constant, then the change is known as isochoric process.

In this process *P* and *T* changes but *V* = constant. Hence Gay-lussac's law is obeyed in this process.

(1) **Equation of state**

From ideal gas equation *PV* = μRT

If volume remains constant $P \propto T$ or $\dfrac{P_1}{T_1} = \dfrac{P_2}{T_2}$ = constant



(2) **Indicator diagram :** Graph I and II represent isometric decrease in pressure at volume V_1 and isometric increase in pressure at volume V_2 respectively and slope of indicator diagram $\frac{dP}{dV} = \infty$

(3) **Specific heat :** Specific heat of gas during isochoric process $C_V = \frac{f}{2} R$

(4) **Bulk modulus of elasticity :** $K = \frac{\Delta P}{-\Delta V / V} = \frac{\Delta P}{0} = \infty$

(5) **Work done in isobaric process :** $\Delta W = P\Delta V = P[V_f - V_i]$ [As V = 0]

$\therefore$ $\Delta W = 0$

(6) **FLTD in isochoric process** : $\Delta Q = \Delta U + \Delta W = \Delta U$ [As ΔW = 0]

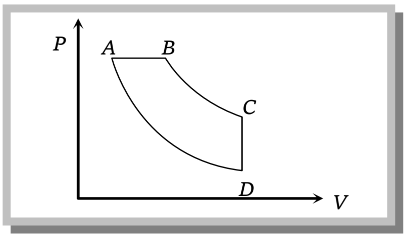
$$\Delta Q = \mu C_V \, \Delta T = \mu \frac{R}{\gamma - 1} \Delta T = \frac{P_f V_f - P_i V_i}{\gamma - 1}$$

Note : ☐ Isometric expansion of the pressure of a gas is given by $P_t = P_0(1 + \gamma_P t)$

where $\gamma_P = \left(\frac{1}{273}\right) per°C =$ coefficient of pressure expansion.

Sample problems based on Isochoric process

Problem **44.** In pressure-volume diagram given below, the isochoric, isothermal, and isobaric parts respectively, are **[Manipal MEE 1995]**



(a) *BA*, *AD*, *DC*

(b) *DC*, *CB*, *BA*

(c) *AB*, *BC*, *CD*

(d) *CD*, *DA*, *AB*

Solution : (d) Process *CD* is isochoric as volume is constant, Process *DA* is isothermal as temperature constant and Process *AB* is isobaric as pressure is constant.

Problem **45.** Molar specific heat of oxygen at constant pressure $C_p = 7.2\, cal/mol/°C$ and R = 8.3 *J/mol/K*. At constant volume, 5 *mol* of oxygen is heated from 10°*C* to 20°*C*, the quantity of heat required is approximately

[MP PMT 1987]

(a) 25 *cal* (b) 50 *cal* (c) 250 *cal* (d) 500 *cal*

Solution : (c) By Mayer's formula $C_v = C_p - R = 7.2 - 2 \approx 5\, cal/mol\, °C$

At constant volume $\Delta Q = \mu c_v \Delta T = 5 \times 5 \times 10 = 250 \; cal$

13.10 Reversible and Irreversible Process

(1) **Reversible process :** A reversible process is one which can be reversed in such a way that all changes occurring in the direct process are exactly repeated in the opposite order and inverse sense and no change is left in any of the bodies taking part in the process or in the surroundings. For example if heat is absorbed in the direct process, the same amount of heat should be given out in the reverse process, if work is done on the working substance in the direct process then the same amount of work should be done by the working substance in the reverse process. The conditions for reversibility are

(i) There must be complete absence of dissipative forces such as friction, viscosity, electric resistance etc.

(ii) The direct and reverse processes must take place infinitely slowly.

(iii) The temperature of the system must not differ appreciably from its surroundings.

Some examples of reversible process are

(a) All isothermal and adiabatic changes are reversible if they are performed very slowly.

(b) When a certain amount of heat is absorbed by ice, it melts. If the same amount of heat is removed from it, the water formed in the direct process will be converted into ice.

(c) An extremely slow extension or contraction of a spring without setting up oscillations.

(d) When a perfectly elastic ball falls from some height on a perfectly elastic horizontal plane, the ball rises to the initial height.

(e) If the resistance of a thermocouple is negligible there will be no heat produced due to Joule's heating effect. In such a case heating or cooling is reversible. At a junction where a cooling effect is produced due to Peltier effect when current flows in one direction and equal heating effect is produced when the current is reversed.

(f) Very slow evaporation or condensation.

It should be remembered that the conditions mentioned for a reversible process can never be realised in practice. Hence, a reversible process is only an ideal concept. In actual process, there is always loss of heat due to friction, conduction, radiation *etc*.

(2) **Irreversible process :** Any process which is not reversible exactly is an irreversible process. All natural processes such as conduction, radiation, radioactive decay *etc*. are irreversible. All practical processes such as free expansion, Joule-Thomson expansion, electrical heating of a wire are also irreversible. Some examples of irreversible processes are given below

(i) When a steel ball is allowed to fall on an inelastic lead sheet, its kinetic energy changes into heat energy by friction. The heat energy raises the temperature of lead sheet. No reverse transformation of heat energy occurs.

(ii) The sudden and fast stretching of a spring may produce vibrations in it. Now a part of the energy is dissipated. This is the case of irreversible process.

(iii) Sudden expansion or contraction and rapid evaporation or condensation are examples of irreversible processes.

(iv) Produced by the passage of an electric current through a resistance is irreversible.

(v) Heat transfer between bodies at different temperatures is also irreversible.

(vi) Joule-Thomson effect is irreversible because on reversing the flow of gas a similar cooling or heating effect is not observed.

13.11 Cyclic and Non-cyclic Process

A cyclic process consists of a series of changes which return the system back to its initial state.

In non-cyclic process the series of changes involved do not return the system back to its initial state.

(1) In case of cyclic process as $U_f = U_i$

$\therefore\ \Delta U = U_f - U_i = 0$ *i.e.* change in internal energy for cyclic process is zero and also $\Delta U \propto \Delta T$

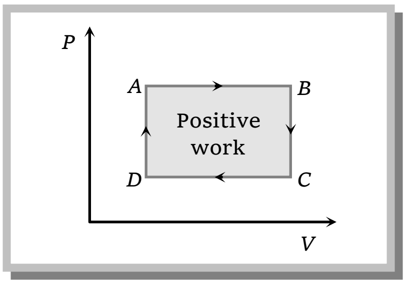
$\therefore\ \Delta T = 0$ *i.e.* temperature of system remains constant.

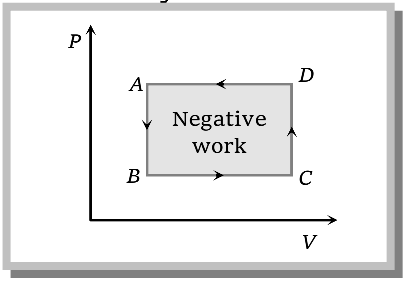
(2) From first law of thermodynamics $\Delta Q = \Delta U + \Delta W$

$\Delta Q = \Delta W$ *i.e.* heat supplied is equal to the work done by the system. [As $\Delta U = 0$]

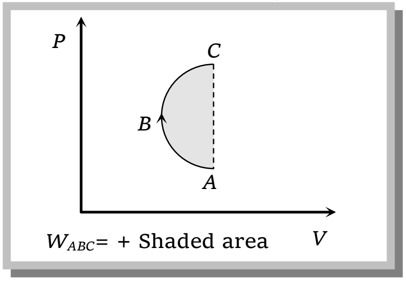
(3) For cyclic process *P*-*V* graph is a closed curve and area enclosed by the closed path represents the work done.

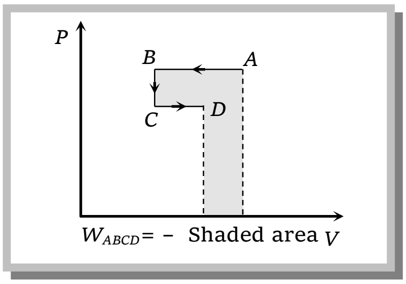
If the cycle is clockwise work done is positive and if the cycle is anticlockwise work done is negative.





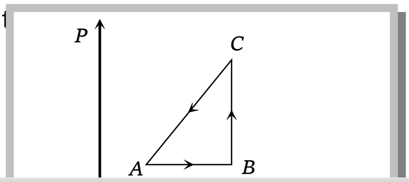
(4) Work done in non cyclic process depends upon the path chosen or the series of changes involved and can be calculated by the area covered between the curve and volume axis on *PV* diagram.





Sample problems based on Cyclic and non-cyclic process

***Problem* 46.** The *P*-*V* diagram of a system undergoing thermodynamic transformation is shown in figure. The work done on the system in going from $A \to B \to C$ is 50 *J* and 20 *cal* heat is given to the system. The change in internal energy bet



(a) 34 *J*

(b) 70 *J*

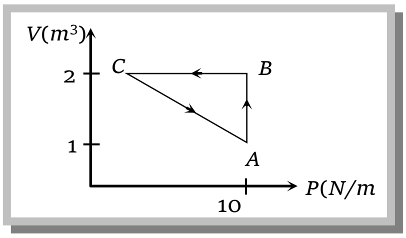
(c) 84 *J*

(d) 134 *J*

Solution : (d) Heat given $\Delta Q = 20\ cal = 20 \times 4.2 = 84\ J$. Work done ΔW = – 50 *J* [As process is anticlockwise]

By first law of thermodynamics $\Rightarrow \Delta U = \Delta Q - \Delta W = 84 - (-50) = 134\ J$

Problem 47. An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 *J*, the work done by the gas in the process $C \to A$ is **[IIT-J**



(a) – 5 *J*

(b) – 10 *J*

(c) – 15 *J*

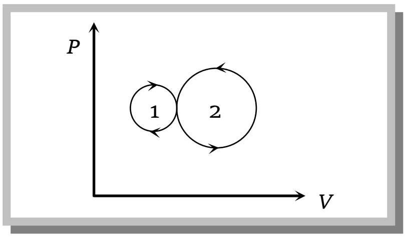
(d) – 20 *J*

Solution : (a) For a cycle process. Total work done $= W_{AB} + W_{BC} + W_{CA}$

$\Rightarrow \frac{1}{2} \times 1.0 \times 10 = 10 \times (2-1) + 0 + W_{CA}$ [W_{BC} = 0 since there is no change in volume along *BC*]

$\Rightarrow 5 J = 10\ J + W_{CA} \Rightarrow W_{CA} = -5\ J$

Problem 48. In the following indicator diagram, the net amount of work done will be



(a) Positive

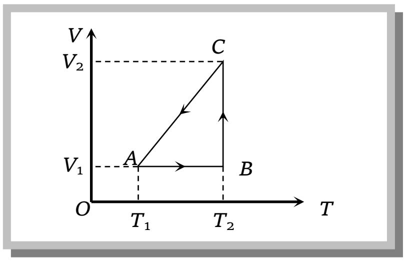
(b) Negative

(c) Zero

(d) Infinity

Solution : (b) Work done during process 1 is positive while during process 2 it is negative. Because process 1 is clockwise while process 2 is anticlockwise. But area enclosed by *P*-*V* graph (*i.e.* work done) in process 1 is smaller so, net work done will be negative.

Problem 49. A cyclic process for 1 mole of an ideal gas is shown in figure in the *V*-*T*, diagram. The work done in *AB*, *BC* and *CA* respectively



(a) $0, RT_2 \ln\left(\frac{V_1}{V_2}\right), R(T_1 - T_2)$

(b) $R(T_1 - T_2), 0, RT_1 \ln \frac{V_1}{V_2}$

(c) $0, RT_2 \ln\left(\frac{V_2}{V_1}\right), R(T_1 - T_2)$

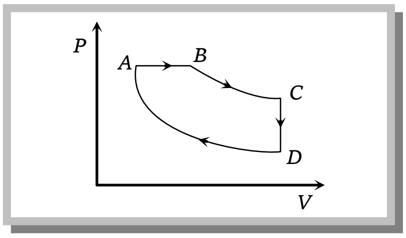
(d) $0, RT_2 \ln\left(\frac{V_2}{V_1}\right), R(T_2 - T_1)$

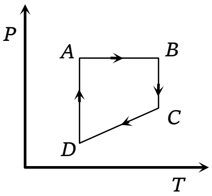
Solution : (c) Process *AB* is isochoric, $\therefore\ W_{AB} = P\,\Delta V = 0$

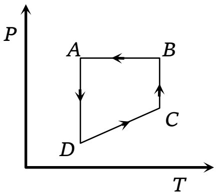
Process *BC* is isothermal $\therefore\ W_{BC} = RT_2.\ln\left(\frac{V_2}{V_1}\right)$

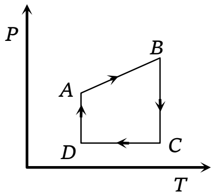
Process *CA* is isobaric $\therefore\ W_{CA} = P\Delta V = R\Delta T = R(T_2 - T_1)$

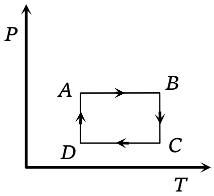
***Problem* 50.** A cyclic process *ABCD* is shown in the figure *P-V* diagram. Which of the following curves represent the same process



(a)


(b)


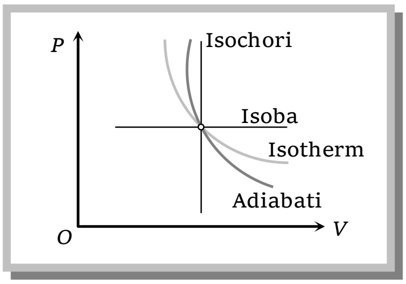
(c)


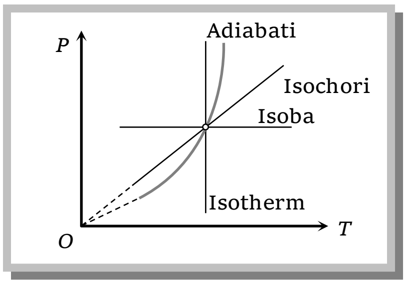
(d)


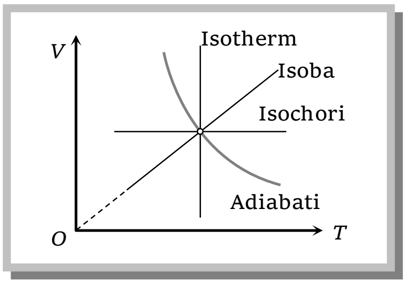
Solution : (a) *AB* is isobaric process, *BC* is isothermal process, *CD* is isometric process and *DA* is isothermal process

These process are correctly represented by graph (a).

13.12 Graphical Representation of Various Processes







13.13 Heat Engine

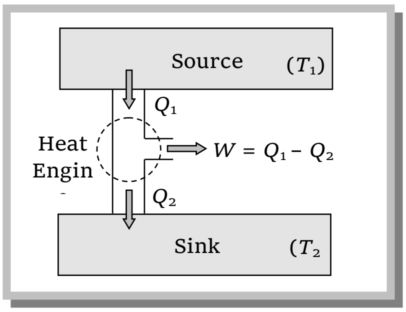
Heat engine is a device which converts heat into work continuously through a cyclic process.

The essential parts of a heat engine are

Source : It is a reservoir of heat at high temperature and infinite thermal capacity. Any amount of heat can be extracted from it.

Working substance : Steam, petrol *etc.*

Sink : It is a reservoir of heat at low temperature and infinite thermal capacity. Any amount of heat can be given to the sink.



The working substance absorbs heat Q_1 from the source, does an amount of work *W*, returns the remaining amount of heat to the sink and comes back to its original state and there occurs no change in its internal energy.

By repeating the same cycle over and over again, work is continuously obtained.

The performance of heat engine is expressed by means of "efficiency" η which is defined as the ratio of useful work obtained from the engine to the heat supplied to it.

$$\eta = \frac{\text{Work done}}{\text{Heat input}} = \frac{W}{Q_1}$$

But by first law of thermodynamics for cyclic process $\Delta U = 0$. $\therefore$ $\Delta Q = \Delta W$ so $W = Q_1 - Q_2$

$$\therefore \qquad \eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

A perfect heat engine is one which converts all heat into work *i.e.* $W = Q_1$ so that $Q_2 = 0$ and hence $\eta = 1$.

But practically efficiency of an engine is always less than 1.

13.14 Refrigerator or Heat Pump

A refrigerator or heat pump is basically a heat engine run in reverse direction.

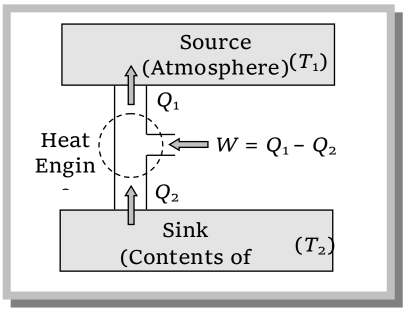
It essentially consists of three parts

Source : At higher temperature T_1.

Working substance : It is called refrigerant liquid ammonia and freon works as a working substance.

Sink : At lower temperature T_2.

The working substance takes heat Q_2 from a sink (contents of refrigerator) at lower temperature, has a net amount of work done W on it by an external agent (usually compressor of refrigerator) and gives out a larger amount of heat Q_1 to a hot body at temperature T_1 (usually atmosphere). Thus, it transfers heat from a cold to a hot body at the expense of mechanical energy supplied to it by an external agent. The cold body is thus cooled more and more.



The performance of a refrigerator is expressed by means of "coefficient of performance" β which is defined as the ratio of the heat extracted from the cold body to the work needed to transfer it to the hot body.

$$i.e. \qquad \beta = \frac{\text{Heat extracted}}{\text{work done}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

$$\therefore \qquad \beta = \frac{Q_2}{Q_1 - Q_2}$$

A perfect refrigerator is one which transfers heat from cold to hot body without doing work *i.e.* $W = 0$ so that $Q_1 = Q_2$ and hence $\beta = \infty$

(1) **Carnot refrigerator**

For Carnot refrigerator $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ $\therefore$ $\frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2}$ or $\frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$

So coefficient of performance $\beta = \frac{T_2}{T_1 - T_2}$

where T_1 = temperature of surrounding, T_2 = temperature of cold body

It is clear that $\beta = 0$ when $T_2 = 0$

i.e. the coefficient of performance will be zero if the cold body is at the temperature equal to absolute zero.

(2) **Relation between coefficient of performance and efficiency of refrigerator**

We know $\beta = \frac{Q_2}{Q_1 - Q_2}$ or $\beta = \frac{Q_2 / Q_1}{1 - Q_2 / Q_1}$ ..... (i)

But the efficiency $\eta = 1 - \frac{Q_2}{Q_1}$ or $\frac{Q_2}{Q_1} = 1 - \eta$ .....(ii)

From (i) and (ii) we get $\beta = \frac{1 - \eta}{\eta}$

13.15 Second Law of Thermodynamics

First law of thermodynamics merely explains the equivalence of work and heat. It does not explain why heat flows from bodies at higher temperatures to those at lower temperatures. It cannot tell us why the converse is possible. It cannot explain why the efficiency of a heat engine is always less than unity. It is also unable to explain why cool water on stirring gets hotter whereas there is no such effect on stirring warm water in a beaker. Second law of thermodynamics provides answers to these questions. Statement of this law is as follows

(1) **Clausius statement :** It is impossible for a self acting machine to transfer heat from a colder body to a hotter one without the aid of an external agency.

From Clausius statement it is clear that heat cannot flow from a body at low temperature to one at higher temperature unless work is done by an external agency. This statement is in fair agreement with our experiences in different branches of physics. For example, electrical current cannot flow from a conductor at lower electrostatic potential to that at higher potential unless an external work is done. Similarly, a body at a lower gravitational potential level cannot move up to higher level without work done by an external agency.

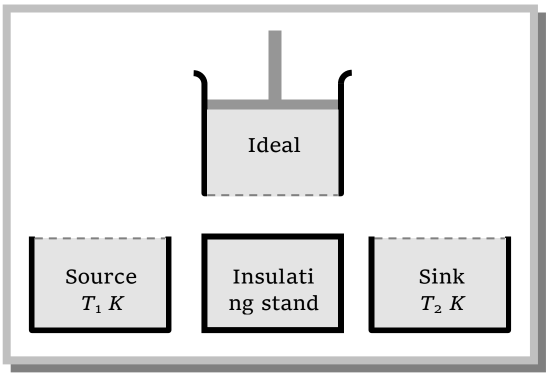
(2) **Kelvin's statement :** It is impossible for a body or system to perform continuous work by cooling it to a temperature lower than the temperature of the coldest one of its surroundings. A Carnot engine cannot work if the source and sink are at the same temperature because work done by the engine will result into cooling the source and heating the surroundings more and more.

(3) **Kelvin-Planck's statement :** It is impossible to design an engine that extracts heat and fully utilises into work without producing any other effect.

From this statement it is clear that any amount of heat can never be converted completely into work. It is essential for an engine to return some amount of heat to the sink. An engine essentially requires a source as well as sink. The efficiency of an engine is always less than unity because heat cannot be fully converted into work.

13.16 Carnot Engine

Carnot designed a theoretical engine which is free from all the defects of a practical engine. This engine cannot be realised in actual practice, however, this can be taken as a standard against which the performance of an actual engine can be judged.



It consists of the following parts

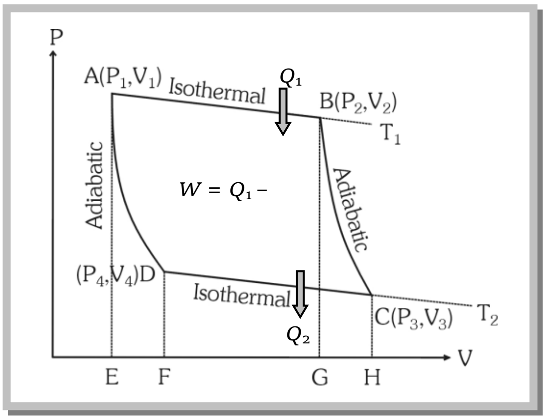
(i) A cylinder with perfectly non-conducting walls and a perfectly conducting base containing a perfect gas as working substance and fitted with a non-conducting frictionless piston

(ii) A source of infinite thermal capacity maintained at constant higher temperature T_1.

(iii) A sink of infinite thermal capacity maintained at constant lower temperature T_2.

(iv) A perfectly non-conducting stand for the cylinder.

(1) **Carnot cycle :** As the engine works, the working substance of the engine undergoes a cycle known as Carnot cycle. The Carnot cycle consists of the following four strokes



(i) First stroke (Isothermal expansion) (curve *AB*) :

The cylinder containing ideal gas as working substance allowed to expand slowly at this constant temperature T_1.

Work done = Heat absorbed by the system

$$W_1 = Q_1 = \int_{V_1}^{V_2} P\,dV = RT_1 \log_e\left(\frac{V_2}{V_1}\right) = \text{Area } ABGE$$

(ii) Second stroke (Adiabatic expansion) (curve *BC*) :

The cylinder is then placed on the non conducting stand and the gas is allowed to expand adiabatically till the temperature falls from T_1 to T_2.

$$W_2 = \int_{V_2}^{V_3} P\,dV = \frac{R}{(\gamma - 1)}[T_1 - T_2] = \text{Area } BCHG$$

(iii) Third stroke (Isothermal compression) (curve *CD*) :

The cylinder is placed on the sink and the gas is compressed at constant temperature T_2.

Work done = Heat released by the system

$$W_3 = Q_2 = -\int_{V_3}^{V_4} P\,dV = -RT_2 \log_e \frac{V_4}{V_3} = RT_2 \log_e \frac{V_3}{V_4} = \text{Area } CDFH$$

(iv) Fourth stroke (adiabatic compression) (curve *DA*) : Finally the cylinder is again placed on non-conducting stand and the compression is continued so that gas returns to its initial stage.

$$W_4 = -\int_{V_4}^{V_1} P\,dV = -\frac{R}{\gamma-1}(T_2 - T_1) = \frac{R}{\gamma-1}(T_1 - T_2) = \text{Area } ADFE$$

(2) **Efficiency of Carnot cycle :** The efficiency of engine is defined as the ratio of work done to the heat supplied *i.e.* $\eta = \frac{\text{work done}}{\text{Heat input}} = \frac{W}{Q_1}$

Net work done during the complete cycle

$$W = W_1 + W_2 + (-W_3) + (-W_4) = W_1 - W_3 = \text{Area } ABCD \qquad [\text{As } W_2 = W_4]$$

$$\therefore \qquad \eta = \frac{W}{Q_1} = \frac{W_1 - W_3}{W_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{W_3}{W_1} = 1 - \frac{Q_2}{Q_1}$$

or $$\eta = 1 - \frac{RT_2 \log_e (V_3/V_4)}{RT_1 \log_e (V_2/V_1)}$$

Since points *B* and *C* lie on same adiabatic curve $\therefore \; T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$ or $\frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1}$ .....(i)

Also point *D* and *A* lie on the same adiabatic curve $\therefore \; T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$ or $\frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$ .....(ii)

From (i) and (ii) $\frac{V_3}{V_2} = \frac{V_4}{V_1}$ or $\frac{V_3}{V_4} = \frac{V_2}{V_1} \Rightarrow \log_e\left(\frac{V_3}{V_4}\right) = \log_e\left(\frac{V_2}{V_1}\right)$

So efficiency of Carnot engine $\eta = 1 - \frac{T_2}{T_1}$

(i) Efficiency of a heat engine depends only on temperatures of source and sink and is independent of all other factors.

(ii) All reversible heat engines working between same temperatures are equally efficient and no heat engine can be more efficient than Carnot engine (as it is ideal).

(iii) As on Kelvin scale, temperature can never be negative (as 0 *K* is defined as the lowest possible temperature) and T_1 and T_2 are finite, efficiency of a heat engine is always lesser than unity, *i.e.*, whole of heat can never be converted into work which is in accordance with second law.

Note : ❑ The efficiency of an actual engine is much lesser than that of an ideal engine. Actually the practical efficiency of a steam engine is about (8-15)% while that of a petrol engine is 40%. The efficiency of a diesel engine is maximum and is about (50-55)%.

(3) **Carnot theorem :** The efficiency of Carnot’s heat engine depends only on the temperature of source (T_1) and temperature of sink (T_2), *i.e.*, $\eta = 1 - \frac{T_2}{T_1}$.

Carnot stated that no heat engine working between two given temperatures of source and sink can be more efficient than a perfectly reversible engine (Carnot engine) working between the same two temperatures. Carnot's reversible engine working between two given temperatures is considered to be the most efficient engine.

13.17 Difference Between Petrol Engine And Diesel Engine

Petrol engine	Diesel engine
(i) Working substance is a mixture of petrol vapour and air.	(i) Working substance in this engine is a mixture of diesel vapour and air.
(ii) Efficiency is smaller (~47%).	(ii) Efficiency is larger (~55%).
(iii) It works with a spark plug.	(iii) It works with an oil plug.
(iv) It is associated with the risk of explosion, because petrol vapour and air is compressed. So, low compression ratio is kept.	(iv) No risk of explosion, because only air is compressed. Hence compression ratio is kept large.
(v) Petrol vapour and air is created with spark plug.	Spray of diesel is obtained through the jet.

Sample problems based on Second law of thermodynamic

Problem **51.** If the door of a refrigerator is kept open, then which of the following is true

[BHU 2001; JIPMER 2002; AIEEE 2002; CPMT 2003]

(a) Room is cooled
(b) Room is heated
(c) Room is either cooled or heated
(d) Room is neither cooled nor heated

Solution : (b) In a refrigerator, the working substance takes heat Q_2 from the sink at lower temperature T_2 and gives out a larger amount of heat Q_1 to a hot body at higher temperature T_1. Therefore the room gets heated if the door of a refrigerator is kept open.

Problem **52.** The coefficient of performance of a Carnot refrigerator working between 30°C and 0°C is **[UPSEAT**

(a) 10
(b) 1
(c) 9
(d) 0

Solution : (c) Coefficient of performance of a Carnot refrigerator working between 30°C and 0°C is

$$\beta = \frac{T_2}{T_1 - T_2} = \frac{273\,°C}{303\,°C - 273\,°C} = \frac{273\,°C}{30\,°C} \approx 9$$

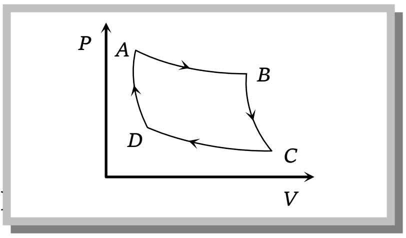
Problem **53.** A Carnot engine working between 300 K and 600 K has work output of 800 J per cycle. What is amount of heat energy supplied to the engine from source per cycle

(a) 1800 J/cycle
(b) 1000 J/cycle
(c) 2000 J/cycle
(d) 1600 J/cycle

Solution : (d) Efficiency of Carnot engine $\eta = \frac{T_1 - T_2}{T_1} \;\; \frac{600 - 300}{600} = \frac{1}{2}$

Again $\eta = \dfrac{\text{Work done}}{\text{Heat input}}$ $\quad\therefore$ Heat input $= \dfrac{\text{Work done}}{\eta} = \dfrac{800}{1/2} = 1600\ J.$

Problem **54.** Carnot cycle (reversible) of a gas represented by a Pressure-Volume curve is shown in the diagram



Consider the following statements

I. Area *ABCD* = Work done on the gas

II. Area *ABCD* = Net heat absorbed

III. Change in the internal energy in cyc

Which of these are correct

(a) I only (b) II only (c) II and III (d) I, II and III

Solution : (c) Work done by the gas (as cyclic process is clockwise) $\therefore \Delta W$ = Area *ABCD*

So from the first law of thermodynamics ΔQ (net heat absorbed) = ΔW = Area *ABCD*

As change in internal energy in cycle $\Delta U = 0$.

Problem **55.** A Carnot engine takes 103 *kcal* of heat from a reservoir at 627°*C* and exhausts it to a sink at 27°*C*. The efficiency of the engine will be

(a) 22.2% (b) 33.3% (c) 44.4% (d) 66.6%

Solution : (d) Efficiency of Carnot engine $= \dfrac{T_1 - T_2}{T_1} = \dfrac{900 - 300}{900} = \dfrac{6}{9}$ or 66.6%.

13.18 Entropy

Entropy is a measure of disorder of molecular motion of a system. Greater is the disorder, greater is the entropy.

The change in entropy *i.e.* $dS = \dfrac{\text{Heat absorbed by system}}{\text{Absolute temperature}}$ or $dS = \dfrac{dQ}{T}$

The relation is called the mathematical form of Second Law of Thermodynamics.

Important points

(1) **For solids and liquids**

(i) When heat given to a substance changes its state at constant temperature, then change in entropy

$$dS = \frac{dQ}{T} = \pm \frac{mL}{T}$$

where positive sign refers to heat absorption and negative sign to heat evolution.

(ii) When heat given to a substance raises its temperature from T_1 to T_2, then change in entropy

$$dS = \int \frac{dQ}{T} = \int_{T_1}^{T_2} mc\,\frac{dT}{T} = mc\,\log_e\left(\frac{T_2}{T_1}\right)$$

$$\Rightarrow \qquad \Delta S = 2.303\, mc \log_e\left(\frac{T_2}{T_1}\right).$$

(2) **For a perfect gas :** Perfect gas equation for n moles is $PV = nRT$

$$\Delta S = \int \frac{dQ}{T} = \int \frac{nC_V dT + P\,dV}{T} \qquad \text{[As } dQ = dU + dW\text{\}}$$

$$\Rightarrow \qquad \Delta S = \int \frac{nC_V dT + \frac{nRT}{V}dV}{T} = nC_V \int_{T_1}^{T_2} \frac{dT}{T} + nR\int_{V_1}^{V_2} \frac{dV}{V} \qquad \text{[As } PV = nRT\text{]}$$

$$\therefore \qquad \Delta S = nC_V \log_e\left(\frac{T_2}{T_1}\right) + nR\log_e\left(\frac{V_2}{V_1}\right)$$

Similarly in terms of T and P $\quad \Delta S = nC_P \log_e\left(\frac{T_2}{T_1}\right) - nR\log_e\left(\frac{P_2}{P_1}\right)$

and in terms of P and V $\quad \Delta S = nC_V \log_e\left(\frac{P_2}{P_1}\right) + nC_P \log_e\left(\frac{V_2}{V_1}\right)$

Sample problems Miscellaneous

***Problem* 56.** An ideal gas expands in such a manner that its pressure and volume can be related by equation PV^2 = constant. During this process, the gas is

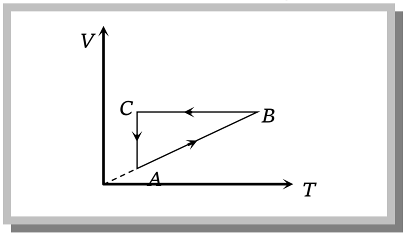
(a) Heated (b) Cooled

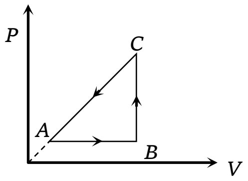
(c) Neither heated nor cooled (d) First heated and then cooled

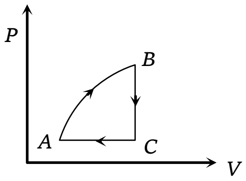
Solution : (b) For an adiabatic expansion $PV^{\gamma} = \text{constant}$ and for the given process PV^2 = constant

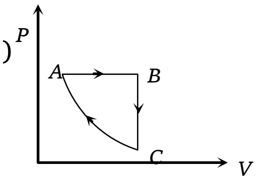
$\therefore$ It is also an adiabatic expansion and during adiabatic expansion the gas is cooled.

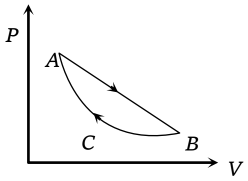
***Problem* 57.** A cyclic process $ABCA$ is shown in the V-T diagram. Process on the P-V diagram is



(a)


(b)


(c)


(d)


Solution : (c) From the given VT diagram, we can see that

In process AB, $V \propto T$ $\therefore$ Pressure is constant (As quantity of the gas remains same)

In process BC, V = constant and in process CA, T = constant

$\therefore$ These processes are correctly represented on PV diagram by graph (c).